Migdal Effect in Dark Matter Direct Detection Experiments

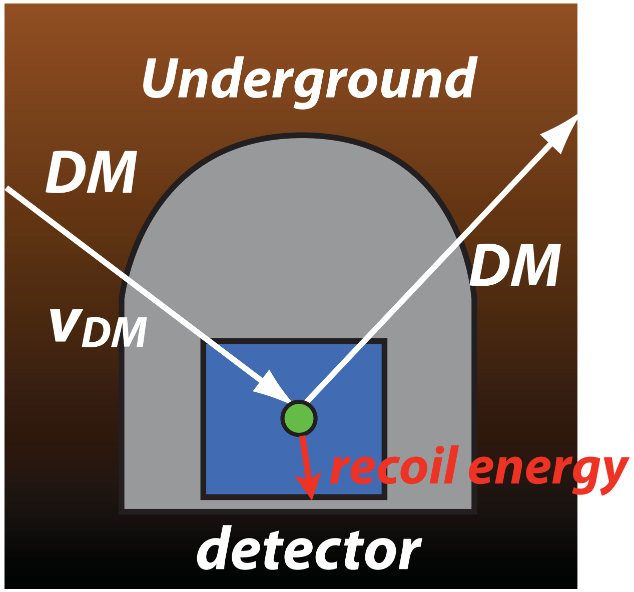
Masahiro Ibe (ICRR)
2020/11/24

Based on a collaboration
arXiv:1707.07258 [with W. Nakano, Y. Shoji, K. Suzuki @ ICRR]

✓ *Dark Matter Direct Detection Experiment*

- ✓ The Earth is immersed in a dark matter halo (ρ_{DM} ~ ***0.3-0.6 GeV/cm³***)
- ✓ Dark Matter in such a halo has a velocity distribution (***<v_{DM}>~220km/s***)
- ✓ The Sun moves at a speed of ***220 km/s*** around the Galaxy.

(The Earth moves around the Sun with a speed of ***30 km/s***)

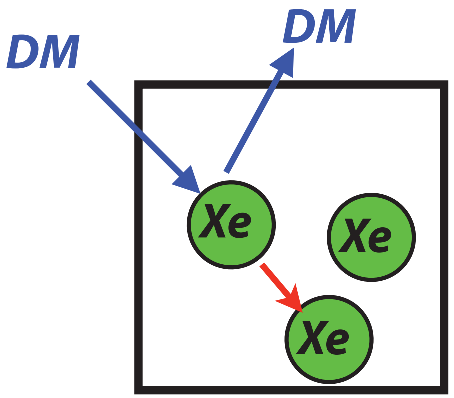


- ✓ Dark matter scatters a nucleus of the detector material and deposits ***recoil energy***.

↓

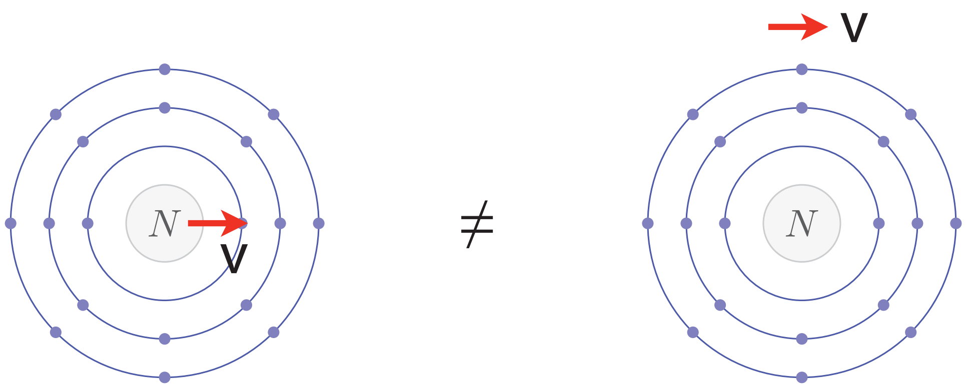
- ✓ The recoil energy is detected through ***ionization***, ***scintillation***, and the production of ***heat*** in the detectors.

✓ *What is missing in the conventional analysis?*



In conventional analysis, the ***recoiled nucleus*** is treated as a ***recoiled neutral atom***.

- ✓ In reality, it takes some time for the electrons to catch up…

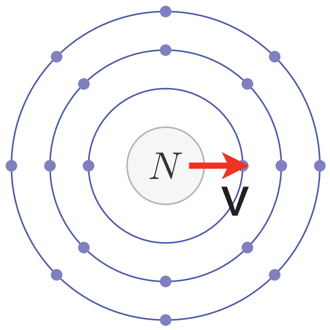


- ✓ The process to catch up causes electron excitations/ionizations!

→ ***Migdal Effect ! [1939, Migdal]***

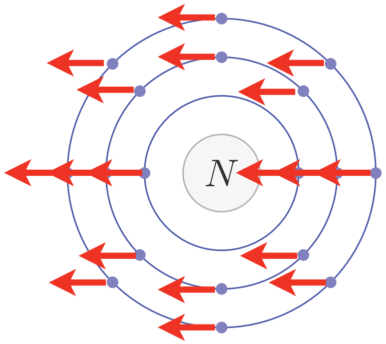
['05 Vergados&Ejiri, '07 Bernabei et al. Application to DM detection]

✔ *Migdal's approach*



Just after the nuclear recoil, we assume only the nucleus is moving while the electron cloud is left behind.

(The electron clouds are no more in the energy eigenstates.)

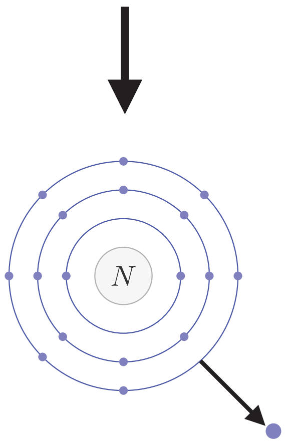


Take the rest frame of the nucleus by the Galilei transformation.

In this frame, the wave function of the electron cloud looks like :

Electron wave function in the initial state e.g. the ground state.

$$|\Phi'_{ec}\rangle = e^{-im_e \sum_i \mathbf{v}\cdot\hat{\mathbf{x}}_i}|\Phi_{ec}\rangle$$



The probability of the excitation/ionization is given by

$$\mathcal{P} = |\langle\Phi^F_{ec}|\Phi'_{ec}\rangle|^2 = |\langle\Phi^F_{ec}|e^{-im_e \sum_i \mathbf{v}\cdot\hat{\mathbf{x}}_i}|\Phi_{ec}\rangle|^2$$

✓ *Disadvantage of the Migdal Approach*

- ✓ The nuclear scattering and the electron excitations/ionizations are treated separately.
  - ✓ Energy Momentum Conservation is not clear…
  - ✓ Where does the electron get energy & momentum?
  - ✓ It is not clear whether the electron excitation energy can be larger than the recoil energy or not.

→ ***It is important to reformulate the Migdal effects in a more coherent way !***

Reformulation of the Migdal Effect

- Migdal's approach

Initial state of the DM scattering : ***(DM plane wave) x (Nucleus plane wave)***

Final state of the DM scattering : ***(DM plane wave) x (Nucleus plane wave)***

Migdal Effect = ***Final state effects***

The Migdal Effect is treated separately from the nuclear scattering

- New approach

Initial state of the DM scattering : ***(DM plane wave) x (Atomic plane wave)***

Final state of the DM scattering : ***(DM plane wave) x (Atomic plane wave)***

The Migdal Effect is automatically taken into account !

How do we construct the plane wave function of the atoms?

Construction of the atomic plane wave

- Hamiltonian of an ***isolated*** atomic system (neutral atom)

$$\hat{H}_A \simeq \frac{\hat{\mathbf{p}}_N^2}{2m_N} + \hat{H}_{ec}(\hat{\mathbf{x}}_N) = \frac{\hat{\mathbf{p}}_N^2}{2m_N} + \sum_i^{N_e} \frac{\hat{\mathbf{p}}_i^2}{2m_e} + V(\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_N)$$

[***V*** are Coulomb forces between the nucleus-electron and the electron-electron]

- Energy eigenstate of the total atomic system (***E_A*** : non-relativistic energy)

$$\left(\frac{\hat{\mathbf{p}}_N^2}{2m_N} + \hat{H}_{ec}(\mathbf{x}_N)\right)\Psi_E(\mathbf{x}_N, \{\mathbf{x}\}) = E_A\,\Psi_E(\mathbf{x}_N, \{\mathbf{x}\})$$

- The approximated energy eigenstate of the atom ***at rest***.

Electron Cloud Energy Eigenstate for a "fixed" ***x_N*** .

$$\hat{H}_{ec}(\mathbf{x}_N)\Phi_{ec}(\{\mathbf{x}\}|\mathbf{x}_N) = E_{ec}(\mathbf{x}_N)\Phi_{ec}(\{\mathbf{x}\}|\mathbf{x}_N)$$

Electron could system does not depend on ***where the Nucleus is.***

$$E_{ec}(\mathbf{x}_N) = E_{ec}\ , \qquad \Phi_{E_{ec}}(\{\mathbf{x}\}|\mathbf{x}_N) = \Phi_{E_{ec}}(\{\mathbf{x} - \mathbf{x}_N\})$$

Born-Oppenheimer approximation !

$$\Psi_{E_A}^{(\mathrm{rest})}(\mathbf{x}_N, \{\mathbf{x}\}) \equiv \Phi_{E_{ec}}(\{\mathbf{x} - \mathbf{x}_N\})$$

$E_A = E_{ec}$

Construction of the atomic plane wave

- Is the Born-Oppenheimer approximation OK ?

$$\left(\frac{\hat{\mathbf{p}}_N^2}{2m_N} + \hat{H}_{ec}(\mathbf{x}_N)\right)\Psi_E(\mathbf{x}_N,\{\mathbf{x}\}) = E_A\,\Psi_E(\mathbf{x}_N,\{\mathbf{x}\})$$

$$\left\langle \frac{\hat{\mathbf{p}}_N^2}{2m_N} \right\rangle \sim \frac{m_e}{m_N} \times E_{ec} \qquad \left(\hat{\mathbf{p}}_N \Phi_{E_c}(\{\mathbf{x}-\mathbf{x}_N\}) = -\sum_i \hat{\mathbf{p}}_i \Phi_{E_c}(\{\mathbf{x}-\mathbf{x}_N\})\right)$$

Kinetic energy of the nucleus is negligible!

- Total Energy Eigenstate in the Born-Oppenheimer approximation of the total Atom at rest

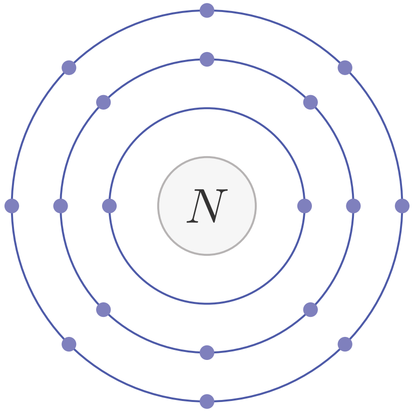
$$\hat{H}_A \Psi_{E_A}^{(\mathrm{rest})}(\mathbf{x}_N,\{\mathbf{x}\}) \simeq E_{ec}\Psi_{E_A}^{(\mathrm{rest})}(\mathbf{x}_N,\{\mathbf{x}\})\ .$$

$$\Psi_{E_A}^{(\mathrm{rest})}(\mathbf{x}_N,\{\mathbf{x}\}) \equiv \Phi_{E_{ec}}(\{\mathbf{x}-\mathbf{x}_N\})$$

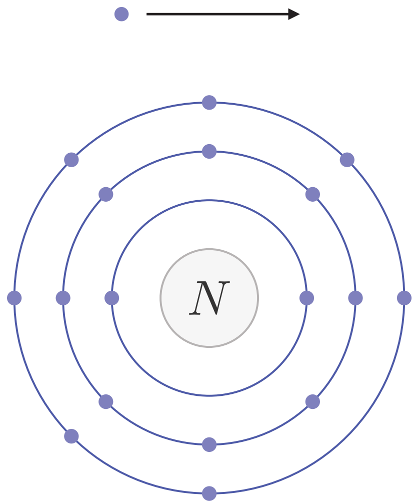
✔ *Construction of the atomic plane wave*

The electrons are not necessarily bounded by the nucleus coulomb force !

$$\Psi_{E_A}^{(\mathrm{rest})}(\mathbf{x}_N, \{\mathbf{x}\}) \equiv \Phi_{E_{ec}}(\{\mathbf{x} - \mathbf{x}_N\})$$



All the electrons are bounded by the Coulomb force of the nucleus.



Not all the electrons are bounded by the Coulomb force of the nucleus
= Ionized atom

The EC wave function can be obtained by e.g. Hartree-Fock approximation !

Construction of the atomic plane wave

- The energy eigenstate of the moving atom with a velocity ***v*** can be obtained by the Galilei transformation !

$$\Psi_{E_A}(\mathbf{x}_N, \{\mathbf{x}\}) \simeq e^{i\mathbf{p}_N\cdot\mathbf{x}_N} e^{i\sum_{i=1}^{N_e}\mathbf{q}_e\cdot\mathbf{x}_i} \Psi_{E_A}^{(\mathrm{rest})}(\mathbf{x}_N, \{\mathbf{x}\})$$

$$\mathbf{p}_N = m_N\mathbf{v} \qquad \mathbf{q}_e = m_e\mathbf{v}$$

Atom wave function at rest

$$E_A \simeq E_{ec} + \frac{1}{2}\overline{m}_A v^2 \qquad \overline{m}_A = m_N + N_e m_e$$

Ψ_{EA} is the eigenstate of the energy and the total atomic momentum !

$$\left(\hat{\mathbf{p}}_N + \sum_i^{N_e}\hat{\mathbf{p}}_i\right)\Psi_{E_A}(\mathbf{x}_N, \{\mathbf{x}\}) = (\overline{m}_A\mathbf{v}) \times \Psi_{E_A}(\mathbf{x}_N, \{\mathbf{x}\})$$

Ψ_{EA} describes the plane wave of the atom ! **($\partial_{xN}\psi_{EA}^{(rest)} = -\Sigma\, \partial_{xi}\psi_{EA}^{(rest)}$)**

Ψ_{EA} is not the eigenstates of the momentums of the nucleus and the electrons separately !

"Atomic" Recoil Cross Section

- DM-Nuclear Scattering without scattering in a field theoretical treatment.

Contact interaction : $$\mathcal{L} = \frac{1}{M_*^2}\overline{\psi}_{p,n}\psi_{p,n}\overline{\psi}_{DM}\psi_{DM}$$

Invariant amplitude^2 : $$|\mathcal{M}|^2 = 16\frac{m_N^2 m_{DM}^2}{M_*^4}A^2$$

Cross section : $$\bar{\sigma}_N \simeq \frac{1}{16\pi}\frac{|\mathcal{M}|^2}{(m_N + m_{DM})^2} \simeq \frac{1}{\pi}\frac{\mu_N^2}{M_*^4}A^2$$

- Nuclear Scattering is reproduced by the point-like interaction potential in QM.

$$\hat{H} = \hat{H}_0 + \hat{V}_{\rm int} \ ,$$
$$\hat{H}_0 = \frac{\hat{\mathbf{p}}_N^2}{2m_N} + \frac{\hat{\mathbf{p}}_{DM}^2}{2m_{DM}} + \hat{V}_{\rm int} \ ,$$
$$\hat{V}_{\rm int} = \frac{-\mathcal{M}}{4m_N m_{DM}}\delta^3(\mathbf{x}_N - \mathbf{x}_{DM})$$

$$|\mathcal{M}|^2 = 16\frac{m_N^2 m_{DM}^2}{M_*^4}A^2$$

Wave Function : [Nuclear Plane Wave] x [DM Plane Wave]

$$\psi_I(\mathbf{x}_N, \mathbf{x}_{DM}) = \sqrt{2m_N}\, e^{i\mathbf{p}_N^I\cdot\mathbf{x}_N} \times \sqrt{2m_{DM}}\, e^{i\mathbf{p}_{DM}^I\cdot\mathbf{x}_{DM}}$$
$$\psi_F(\mathbf{x}_N, \mathbf{x}_{DM}) = \sqrt{2m_N}\, e^{i\mathbf{p}_N^F\cdot\mathbf{x}_N} \times \sqrt{2m_{DM}}\, e^{i\mathbf{p}_{DM}^F\cdot\mathbf{x}_{DM}}$$

"Atomic" Recoil Cross Section

- DM-Nuclear Scattering without scattering in a field theoretical treatment.

Contact interaction : $$\mathcal{L} = \frac{1}{M_*^2}\overline{\psi}_{p,n}\psi_{p,n}\overline{\psi}_{DM}\psi_{DM}$$

Invariant amplitude² : $$|\mathcal{M}|^2 = 16\frac{m_N^2 m_{DM}^2}{M_*^4}A^2$$

Cross section : $$\bar{\sigma}_N \simeq \frac{1}{16\pi}\frac{|\mathcal{M}|^2}{(m_N+m_{DM})^2} \simeq \frac{1}{\pi}\frac{\mu_N^2}{M_*^4}A^2$$

- Nuclear Scattering is reproduced by the point-like interaction potential.

$$\hat{H} = \hat{H}_0 + \hat{V}_{\rm int} \ ,$$
$$\hat{H}_0 = \frac{\hat{\mathbf{p}}_N^2}{2m_N} + \frac{\hat{\mathbf{p}}_{DM}^2}{2m_{DM}} + \hat{V}_{\rm int} \ ,$$
$$\hat{V}_{\rm int} = \frac{-\mathcal{M}}{4m_N m_{DM}}\delta^3(\mathbf{x}_N - \mathbf{x}_{DM})$$

$$|\mathcal{M}|^2 = 16\frac{m_N^2 m_{DM}^2}{M_*^4}A^2$$

Born Approximation

$$\rightarrow \quad T_{FI} = \mathcal{M} \times i(2\pi)^4\delta(E_N^F + E_{DM}^F - E_N^I - E_{DM}^I)\delta^3(\mathbf{p}_N^F + \mathbf{p}_{DM}^F - \mathbf{p}_N^I - \mathbf{p}_{DM}^I)$$

(with the asymptotic Nucleus plane waves)

"Atomic" Recoil Cross Section

- Atomic Scattering via the contact DM-nuclear interaction term :

$$\hat{H}_{\rm tot} = \hat{H}_A + \frac{\hat{\mathbf{p}}_{DM}^2}{2m_{DM}} + \hat{V}_{\rm int}$$

$$\hat{V}_{\rm int} = \frac{-\mathcal{M}}{4m_N m_{DM}} \delta^3(\mathbf{x}_N - \mathbf{x}_{DM})$$

Initial: $\Psi_I(\mathbf{x}_N, \{\mathbf{x}\}, \mathbf{x}_{DM}) = \sqrt{2m_N}\,\underline{\Psi_{E_A^I}(\mathbf{x}_N, \{\mathbf{x}\})} \times \sqrt{2m_{DM}}\, e^{i\mathbf{p}_{DM}^I\cdot\mathbf{x}_{DM}}$

Final: $\Psi_F(\mathbf{x}_N, \{\mathbf{x}\}, \mathbf{x}_{DM}) = \sqrt{2m_N}\,\underline{\Psi_{E_A^F}(\mathbf{x}_N, \{\mathbf{x}\})} \times \sqrt{2m_{DM}}\, e^{i\mathbf{p}_{DM}^F\cdot\mathbf{x}_{DM}}$

(Atomic plane wave)

(The normalization is to conform with ***$<p'|p> = (2E)^{1/2}(2\pi)^{3/2}\delta^3(p'-p)$***)

- We assume that initial sate atom is at rest : ***$p^I_A = 0$***.

Initial: $E_I = E_{ec}^I + \frac{{\mathbf{p}_{DM}^I}^2}{2m_{DM}}$,

Final: $E_F = E_{ec}^F + \frac{m_A}{2}{v_F}^2 + \frac{{\mathbf{p}_{DM}^F}^2}{2m_{DM}}$,

“Atomic” Recoil Cross Section

- Atomic Scattering via the contact DM-nuclear interaction term :

$$\hat{H}_{\rm tot} = \hat{H}_A + \frac{\hat{\mathbf{p}}_{DM}^2}{2m_{DM}} + \hat{V}_{\rm int}$$

$$\hat{V}_{\rm int} = \frac{-\mathcal{M}}{4m_N m_{DM}}\delta^3(\mathbf{x}_N - \mathbf{x}_{DM})$$

$$T_{FI} = \mathcal{M}\times i(2\pi)\delta(E_F - E_I)\int d^3\mathbf{x}_N d^3\mathbf{x}_{DM}\prod_i d^3\mathbf{x}_i\,\delta^3(\mathbf{x}_N - \mathbf{x}_{DM})$$
$$\times\Phi^*_{E^F_{ec}}(\{\mathbf{x}-\mathbf{x}_N\})e^{-i\sum_i \mathbf{q}_e\cdot\mathbf{x}_i}e^{-i\mathbf{p}_N^F\cdot\mathbf{x}_N}\Phi_{E^I_{ec}}(\{\mathbf{x}-\mathbf{x}_N\})e^{-i(\mathbf{p}^F_{DM}-\mathbf{p}^I_{DM})\cdot\mathbf{x}_{DM}}$$
$$= \mathcal{M}\times i(2\pi)^4\delta(E_F - E_I)\delta^3(\overline{m}_A\mathbf{v}_F + \mathbf{p}^F_{DM} - \mathbf{p}^I_{DM})$$
$$\times\int\prod_i d^3\mathbf{x}_i\,\Phi^*_{E^F_{ec}}(\{\mathbf{x}\})e^{-i\sum_i \mathbf{q}_e\cdot\mathbf{x}_i}\Phi_{E^I_{ec}}(\{\mathbf{x}\})\ .$$

(correct energy momentum conservation)

Migdal factor !

By taking the asymptotic states consist of the atomic plane waves, the Migdal factor appears automatically.
The total energy momentum conservation is manifest !

✓ "Atomic" Recoil Cross Section

After phase space integration (center of mass frame):

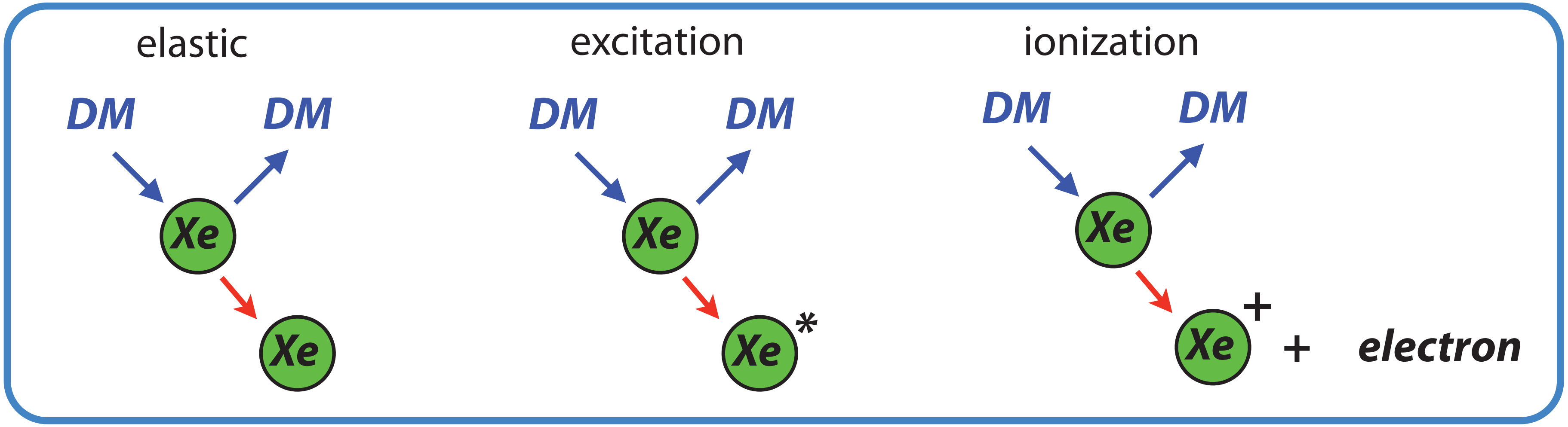
$$\frac{d\sigma}{d\cos\theta_{CM}} \simeq \sum_{E_{ec}^F} \frac{1}{32\pi} \frac{|\mathbf{p}_F|}{(p_A^{I\,0} + p_{DM}^{I\,0})^2 |\mathbf{p}_I|} |F_A(q_A^2)|^2 |\mathcal{M}(q_A^2)|^2 \underline{|Z_{FI}(q_e)|^2} \,.$$

$$Z_{FI}(\mathbf{q}_e) = \int \prod_i d^3\mathbf{x}_i \, \Phi^*_{E_{ec}^F}(\{\mathbf{x}\}) e^{-i\sum_i \mathbf{q}_e\cdot\mathbf{x}_i} \Phi_{E_{ec}^I}(\{\mathbf{x}\})$$

$$\mathbf{p}_{DM}^I = -\mathbf{p}_A^I = \mathbf{p}_I \simeq \mu_N \mathbf{v}_{DM}^I \qquad \text{(CM)}$$

The process is not elastic for $E_{ec}^F \neq E_{ec}^I$!

In CM: $|\mathbf{p}_F|^2 \simeq |\mathbf{p}_I|^2 - 2\mu_N(E_{ec}^F - E_{ec}^I)$ $\qquad v_{DM}^{(th)} = \sqrt{\frac{2(E_{ec}^F - E_{ec}^I)}{\mu_N}}$



✓ The dark matter event rate per unit detector mass

$$\frac{dR}{dE_R dv_{DM}} \simeq \frac{1}{m_A}\frac{\rho_{DM}}{m_{DM}}\frac{d\sigma}{dE_R} v_{DM}\tilde{f}_{DM}(v_{DM}) \ ,$$

$$\simeq \sum_{E_{ec}^F} \frac{1}{2}\frac{\rho_{DM}}{m_{DM}}\frac{1}{\mu_N^2}|F_A(q_A^2)|^2\bar{\sigma}_N \times \underline{|Z_{FI}(q_e)|^2} \times \frac{\tilde{f}(v_{DM})}{v_{DM}}$$

Migdal factor

DM velocity distribution $\int \tilde{f}_{DM}(v_{DM})\, dv_{DM} = 1$

$$E_R \simeq \frac{q_A^2}{2m_A} \simeq \frac{|\mathbf{p}_F|^2 + |\mathbf{p}_I|^2 - 2|\mathbf{p}_I||\mathbf{p}_F|\cos\theta_{CM}}{2m_A}$$

$$\mathbf{p}_{DM}^I = -\mathbf{p}_A^I = \mathbf{p}_I \simeq \mu_N \mathbf{v}_{DM}^I$$

$$|\mathbf{p}_F|^2 \simeq |\mathbf{p}_I|^2 - 2\mu_N(E_{ec}^F - E_{ec}^I)$$

$E_{ec}^F = E_{ec}^I$: nuclear recoil = atomic recoil (conventional dark matter event)

$E_{ec}^F \neq E_{ec}^I$: nuclear recoil = atomic recoil + electric energy injection !

Migdal Effect converts some of the recoil energy into electronic energy !

Single Electron Approximation

- For numerical estimation, we use the Dirac-Hartree-Fock approximation to obtain the electron wave functions .

  Electron wave function ~ Slater determinant of single electrons

- Accordingly, the "atomic plane wave" is also given by a Slater determinant

$$\Psi_{E_A}(\mathbf{x}_N, \{\mathbf{x}\}) \simeq e^{i\mathbf{p}_N\cdot\mathbf{x}_N} \sum_{\sigma\in S_{N_e}} \frac{\mathrm{sgn}(\sigma)}{\sqrt{N_e!}} e^{i\mathbf{q}_e\cdot\mathbf{x}_1} \phi^{\alpha_1}_{o_{\sigma(1)}}(\mathbf{x}_1 - \mathbf{x}_N) e^{i\mathbf{q}_e\cdot\mathbf{x}_2} \phi^{\alpha_2}_{o_{\sigma(2)}}(\mathbf{x}_2 - \mathbf{x}_N) \times \cdots e^{i\mathbf{q}_e\cdot\mathbf{x}_{N_e}} \phi^{\alpha_{N_e}}_{o_{\sigma(N_e)}}(\mathbf{x}_{N_e} - \mathbf{x}_N) \ ,$$

$$ec = \{o_1, o_2, \cdots o_{N_e}\} \qquad o_i = (E_i, \kappa_i, m_i) \qquad \kappa = \mp(j+1/2) \text{ for } j = \ell \pm 1/2.$$

$$\sum_{\alpha=1}^{4} \int d^3\mathbf{x}\, \phi_o(\mathbf{x})^{\alpha *} \phi^{\alpha}_{o'}(\mathbf{x}) = \begin{cases} \delta_{nn'}\delta_{\kappa\kappa'}\delta_{mm'} & \text{(bounded)} \\ (2\pi)\delta(E-E')\delta_{\kappa\kappa'}\delta_{mm'} & \text{(unbounded)} \end{cases}$$

α_i = 1- 4 : Dirac Spinor index

- In this approximation, the Migdal factor is given by the transition late between the single electron orbitals

$$Z_{FI}(\mathbf{q}_e) = \sum_{\sigma\in S_{N_e}} \mathrm{sgn}(\sigma) \prod_{i=1}^{N_e} \sum_{\alpha_i=1}^{4} \int d^3\mathbf{x}_i\, \phi^{\alpha_i *}_{o^F_{\sigma(i)}}(\mathbf{x}_i) e^{\underline{-i\mathbf{q}_e\cdot\mathbf{x}_i}} \phi^{\alpha_i}_{o^I_i}(\mathbf{x}_i)$$

✓ *Single Electron Approximation*

- For a DM-nucleus scattering,

$$q_e = m_e\, q_A/m_A < 10^{-3}\, m_e\, (q_A/100\text{MeV}) \qquad (q_A = \mu_A\, v_{DM})$$

$$\rightarrow q_e x_e << q_e\, x\ (\text{Bhor Radius}) < 1$$

- At the leading order of q_e, ***only one electron*** can be excited/ionized.

  For a given set of the initial orbitals, only one orbital can be different in the final state.

$$ec = \{o_1, \cdots, o_k, \cdots\} \rightarrow ec' = \{o_1, \cdots, o'_k, \cdots\}$$

$$Z_{FI}(\mathbf{q}_e) = z_{\mathbf{q}_e}(E'_k, \kappa'_k, m'_k | E_k, \kappa_k, m_k) = -i \sum_{\alpha_k=1}^{4} \int d^3\mathbf{x}_k\, \phi^{\alpha_k *}_{o'_k}(\mathbf{x}_k)(\mathbf{q}_e \cdot \mathbf{x}_k)\phi^{\alpha_k}_{o_k}(\mathbf{x}_k)$$

$$\sum_F |Z_{FI}|^2 = \underbrace{|Z_{II}|^2}_{\sim 1} + \sum_{n,\ell,n',\ell'} p^d_{q_e}(n\ell \rightarrow n'\ell') + \sum_{n,\ell} \int \frac{dE_e}{2\pi} \frac{d}{dE_e} p^c_{q_e}(n\ell \rightarrow E_e)$$

elastic — **excitation $\propto q_e^2$** — **ionization $\propto q_e^2$**

excitation/ionization rates can be obtained via the wave functions of the single electron orbitals

Numerical Transition Rate (by using Flexible Atomic Code)

Xe ($q_e = m_e \times 10^{-3}$)

Excitation / Ionization

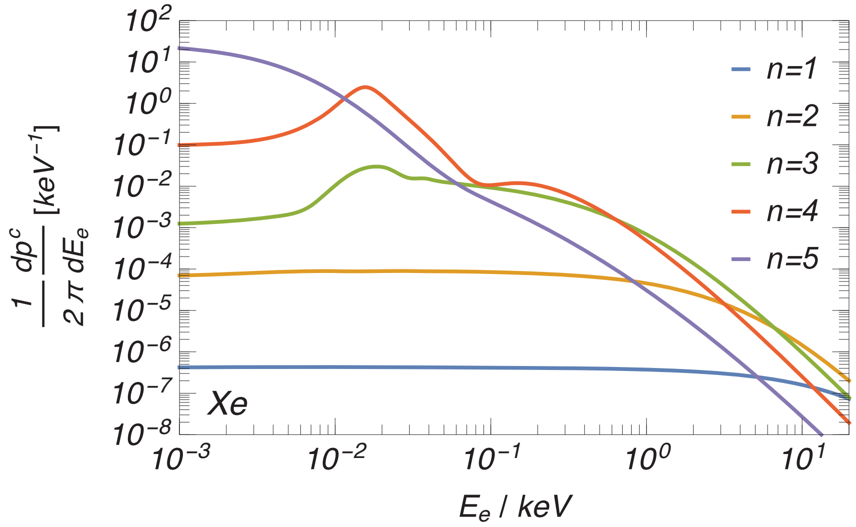
(n,ℓ)	$\mathcal{P}_{\to 4f}$	$\mathcal{P}_{\to 5d}$	$\mathcal{P}_{\to 6s}$	$\mathcal{P}_{\to 6p}$	$E_{n\ell}$ [eV]	$\frac{1}{2\pi}\int dE_e \frac{dp^c}{dE_e}$
1s	–	–	–	7.3×10^{-10}	3.5×10^{4}	4.6×10^{-6}
2s	–	–	–	1.8×10^{-8}	5.4×10^{3}	2.9×10^{-5}
2p	–	3.0×10^{-8}	6.5×10^{-9}	–	4.9×10^{3}	1.3×10^{-4}
3s	–	–	–	2.7×10^{-7}	1.1×10^{3}	8.7×10^{-5}
3p	–	3.4×10^{-7}	4.0×10^{-7}	–	9.3×10^{2}	5.2×10^{-4}
3d	2.3×10^{-9}	–	–	4.3×10^{-7}	6.6×10^{2}	3.5×10^{-3}
4s	–	–	–	3.1×10^{-6}	2.0×10^{2}	3.4×10^{-4}
4p	–	4.1×10^{-8}	3.0×10^{-5}	–	1.4×10^{2}	1.4×10^{-3}
4d	7.0×10^{-7}	–	–	1.5×10^{-4}	6.1×10	3.4×10^{-2}
5s	–	–	–	1.2×10^{-4}	2.1×10	4.1×10^{-4}
5p	–	3.6×10^{-2}	2.1×10^{-2}	–	9.8	1.0×10^{-1}

initial state

(n,ℓ)	$4f$	$5d$	$6s$	$6p$
$E_{n\ell}$[eV]	0.85	1.6	3.3	2.2

(transition is possible only for $|\Delta\ell| = 1$)

ionization spectrum $\propto q_e^2$



- The ionization rate from ***n* = 3** state can be of ***O*(10$^{-(3\text{-}2)}$)**.
  → leading to ***O*(1)keV** electronic energy deposition !
- The rates for the excitation to the higher shells are smaller.

✔ Numerical Transition Rate (by using Flexible Atomic Code)

Xe ($q_e = m_e \times 10^{-3}$)

Excitation — Ionization

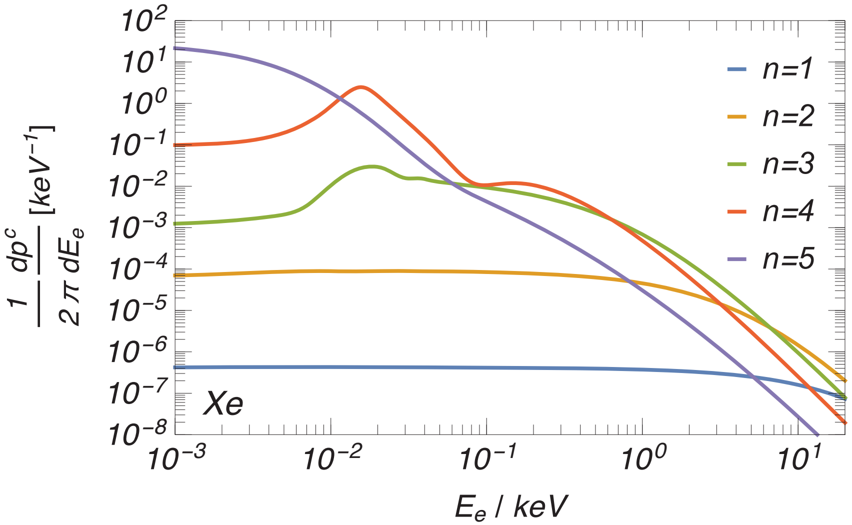
(n,ℓ)	$\mathcal{P}_{\to 4f}$	$\mathcal{P}_{\to 5d}$	$\mathcal{P}_{\to 6s}$	$\mathcal{P}_{\to 6p}$	$E_{n\ell}$ [eV]	$\frac{1}{2\pi}\int dE_e \frac{dp^c}{dE_e}$
1s	–	–	–	7.3×10^{-10}	3.5×10^{4}	4.6×10^{-6}
2s	–	–	–	1.8×10^{-8}	5.4×10^{3}	2.9×10^{-5}
2p	–	3.0×10^{-8}	6.5×10^{-9}	–	4.9×10^{3}	1.3×10^{-4}
3s	–	–	–	2.7×10^{-7}	1.1×10^{3}	8.7×10^{-5}
3p	–	3.4×10^{-7}	4.0×10^{-7}	–	9.3×10^{2}	5.2×10^{-4}
3d	2.3×10^{-9}	–	–	4.3×10^{-7}	6.6×10^{2}	3.5×10^{-3}
4s	–	–	–	3.1×10^{-6}	2.0×10^{2}	3.4×10^{-4}
4p	–	4.1×10^{-8}	3.0×10^{-5}	–	1.4×10^{2}	1.4×10^{-3}
4d	7.0×10^{-7}	–	–	1.5×10^{-4}	6.1×10	3.4×10^{-2}
5s	–	–	–	1.2×10^{-4}	2.1×10	4.1×10^{-4}
5p	–	3.6×10^{-2}	2.1×10^{-2}	–	9.8	1.0×10^{-1}

initial state

(n,ℓ)	$4f$	$5d$	$6s$	$6p$
$E_{n\ell}$[eV]	0.85	1.6	3.3	2.2

(transition is possible only for **|Δℓ| = 1**)

ionization spectrum $\propto q_e^2$



E_e spectrum is purely determined the structure of the electron cloud !

E_e spectrum is independent of the dark matter velocity v_{DM} and m_{DM}.

Rate is proportional to q_e^2

✓ Differential Ionization Event Rate for an Isolated Atom

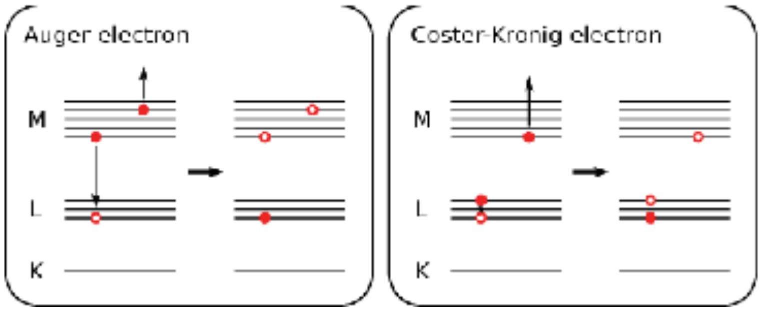
$$\frac{dR}{dE_R\,dE_e\,dv_{DM}} \simeq \frac{dR_0}{dE_R\,dv_{DM}} \times \frac{1}{2\pi}\sum_{n,\ell}\frac{d}{dE_e}p^c_{q_e}(n\ell \to E_e)\ ,$$

$$\frac{dR_0}{dE_R\,dv_{DM}} \simeq \frac{1}{2}\frac{\rho_{DM}}{m_{DM}}\frac{1}{\mu_N^2}|F_A(q_A^2)|^2\bar{\sigma}_N \times \frac{\tilde{f}(v_{DM})}{v_{DM}}\ ,$$

(E_e : free electron kinetic energy)

Ionization = free electron + ion with a core hole

✓ When the core-hole (the vacancy in the inner shell) is created by ionization, the states are de-excited immediately in ***O(10)fs***.



New J.Phys. 15 (2013) 083040

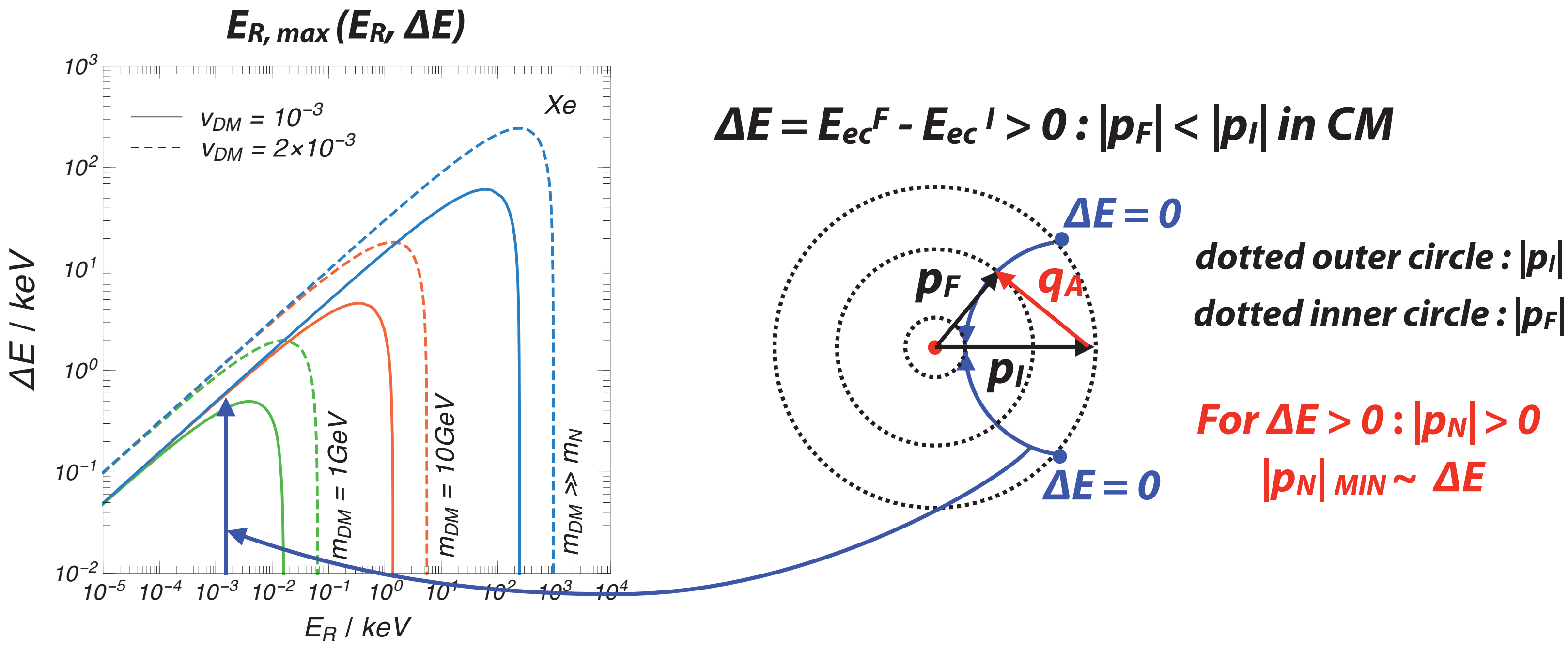
The electron energy and the de-excitation energy are measured simultaneously.

$$E_{EM} = E_e + E_{\rm dex}$$
$$\simeq \Delta E\ = (E^F_{ec} - E^I_{ec})$$

Differential Event Rate with respect to the measurable electric energy

$$\frac{dR}{dE_R\,dE_{EM}\,dv_{DM}} \simeq \frac{dR_0}{dE_R\,dv_{DM}} \times \frac{1}{2\pi}\sum_{n,\ell}\frac{d}{dE_e}p^c_{q_e}(n\ell \to (E_{EM} - E_{\rm dex}))$$

✔ *Kinematical Constraint*



$$\mathbf{p}^I_{DM} = -\mathbf{p}^I_A = \mathbf{p}_I \simeq \mu_N \mathbf{v}^I_{DM} \qquad |\mathbf{p}_F|^2 \simeq |\mathbf{p}_I|^2 - 2\mu_N \Delta E$$

$$E_R \simeq \frac{q_A^2}{2m_A} \simeq \frac{|\mathbf{p}_F|^2 + |\mathbf{p}_I|^2 - 2|\mathbf{p}_I||\mathbf{p}_F|\cos\theta_{CM}}{2m_A}$$

The maximum ***ΔE***

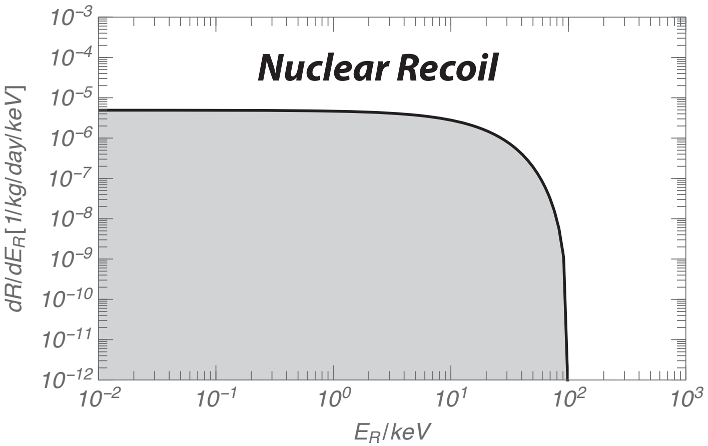
$$\Delta E_{\max} = \frac{m_A}{\mu_N} E_R - \sqrt{2 m_A E_R} v_{\rm DM}$$

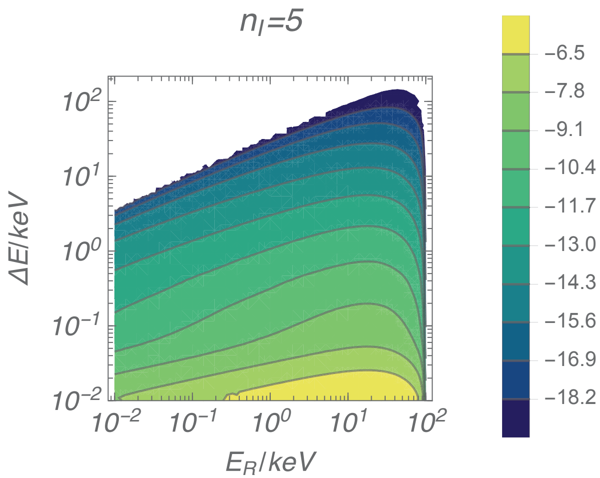
✓ Differential Event Rate for an Isolated Atom

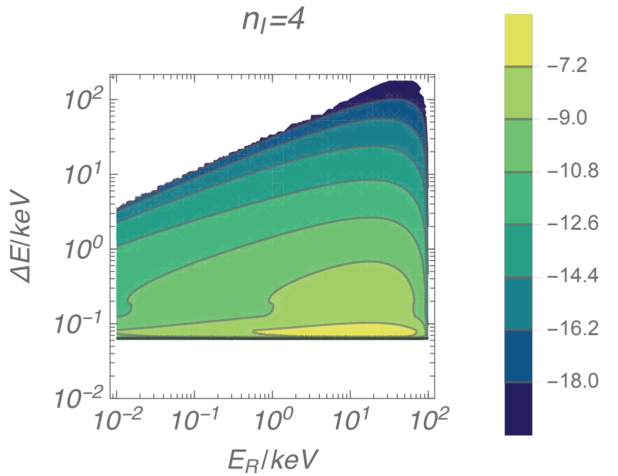
$m_{DM} = 1 TeV$, $\sigma_N = 10^{-45} cm^2$

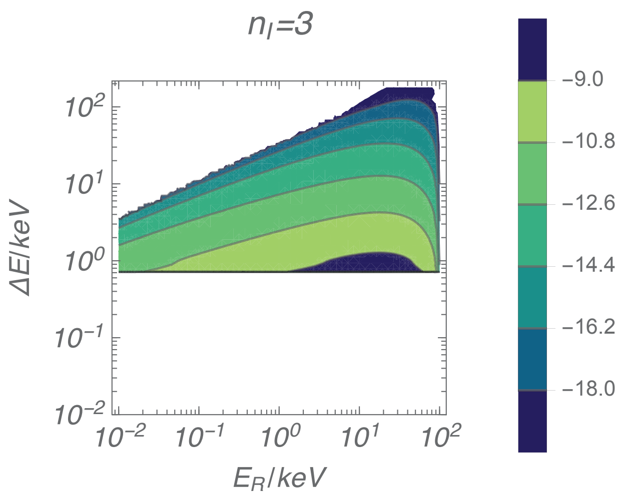
$dR/dE_R d\Delta E$ [1/kg/day/keV/keV] $\Delta E > E_{ion}$

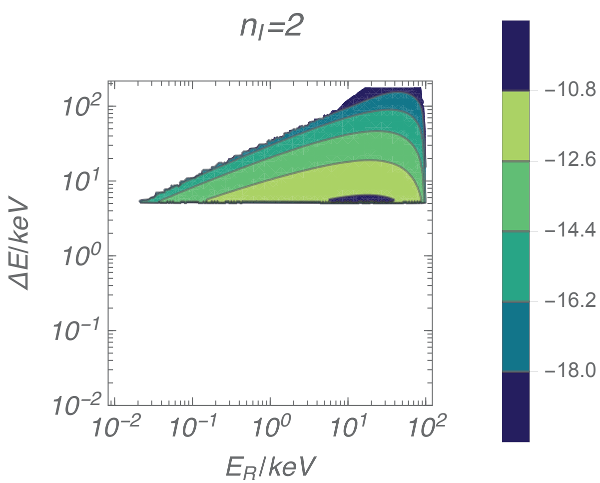
dR/dE_R [1/kg/day/keV]

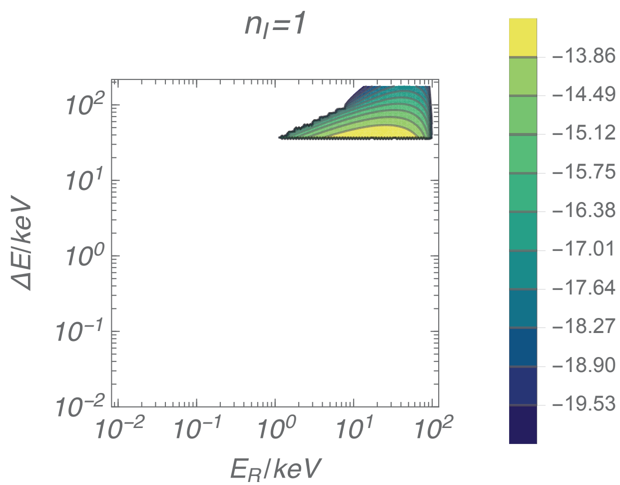












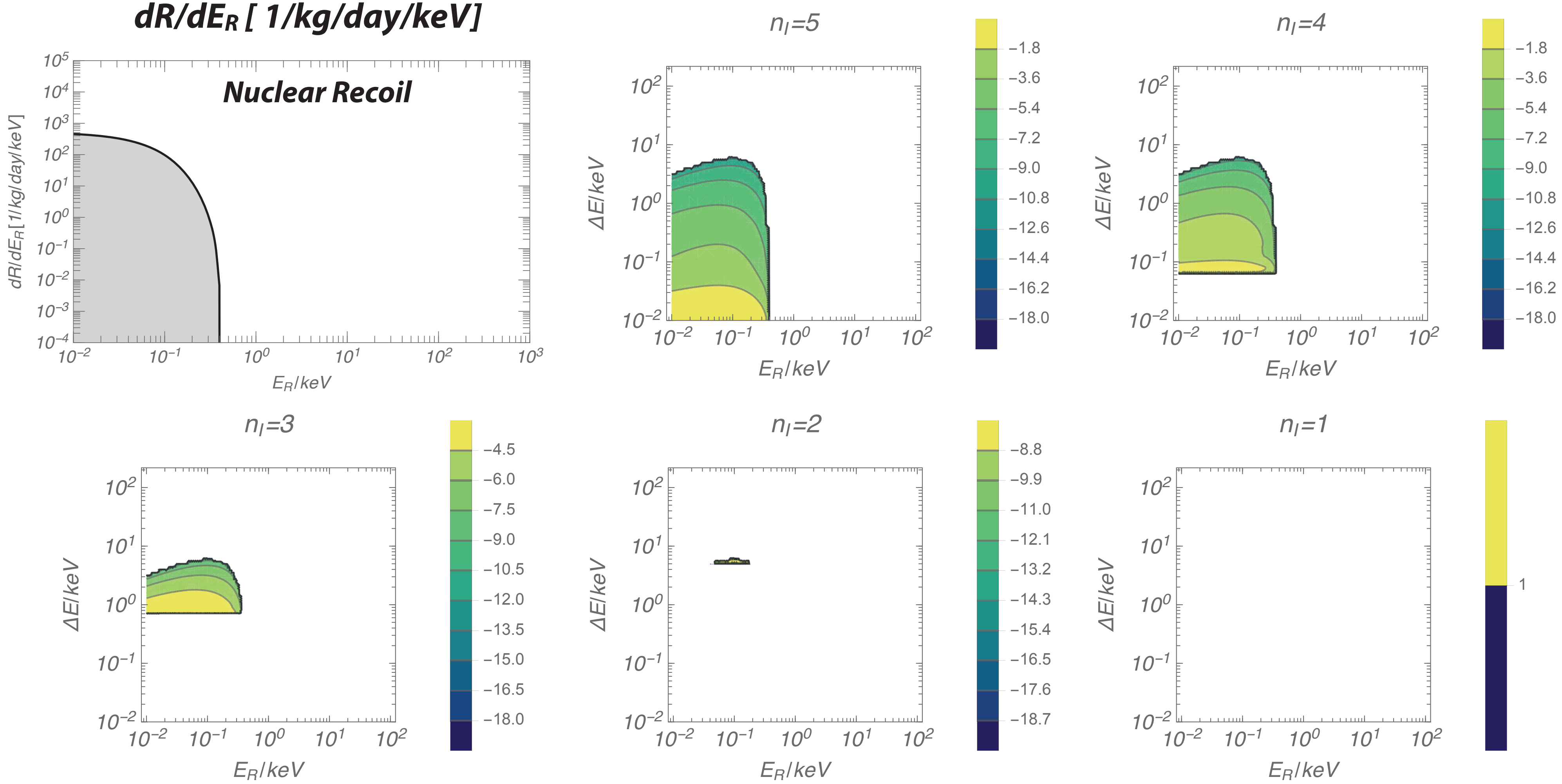
Ionization rate from an outer orbit is higher !

✓ Differential Event Rate for an Isolated Atom

$m_{DM} = 2\text{GeV},\ \sigma_N = 10^{-40}\text{cm}^2$

$dR/dE_R d\Delta E$ [1/kg/day/keV/keV] $\Delta E > E_{ion}$

dR/dE_R [1/kg/day/keV]



Typical ΔE is independent of the DM mass

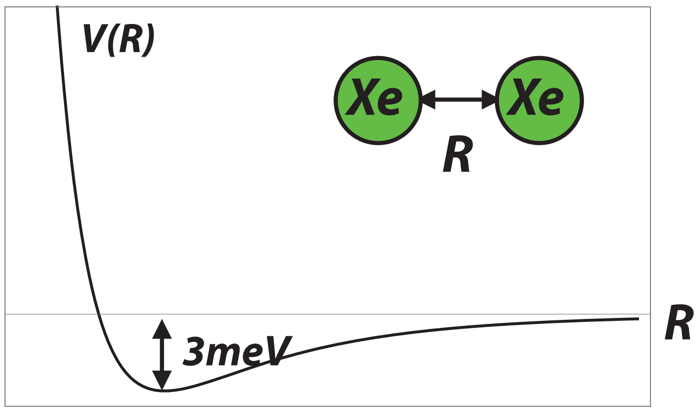
$E_{R,\ MAX}$ is suppressed for a smaller DM

✓ *Implication on Dark Matter Direct Detection Experiments*

- In the detector, the atoms are not isolated .

  e.g.) Typical separation in the liquid Xe ground state ***~ 2 x 10^{-8} cm***

- The wave function of the ***valence (the outermost) electrons*** are affected by the electrons of the neighbor atoms.



2x10^{-8} cm ~ 4 x Bohr radius

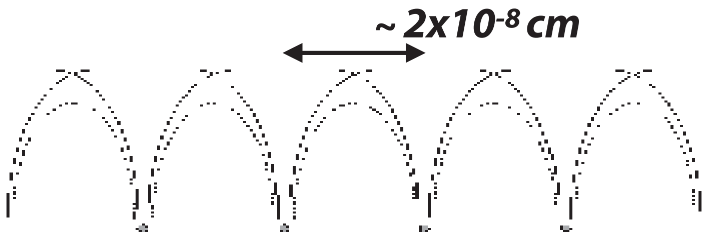
van der Waals force

= deformation of the electron cloud

→ the transition rate from the valence electrons for the isolated atom is not reliable

- Ionization energies are slightly reduced by about ***O(1)eV***

  → the transition rates from the valence electrons for the isolated atom are not reliable



potential of the valence quark

✓ *Implication on Dark Matter Direct Detection Experiments*

Electron Orbits

The number of electrons in a shell for the ground state configurations.

	$1s$	$2s$	$2p$	$3s$	$3p$	$3d$	$4s$	$4p$	$4d$	$4f$	$5s$	$5p$
Na	2	2	6	~~1~~	0	0	0	0	0	0	0	0
Ar	2	2	6	~~2~~	~~6~~	0	0	0	0	0	0	0
Ge	2	2	6	2	6	10	~~2~~	~~2~~	0	0	0	0
I	2	2	6	2	6	10	2	6	10	0	~~2~~	~~5~~
Xe	2	2	6	2	6	10	2	6	10	0	~~2~~	~~6~~

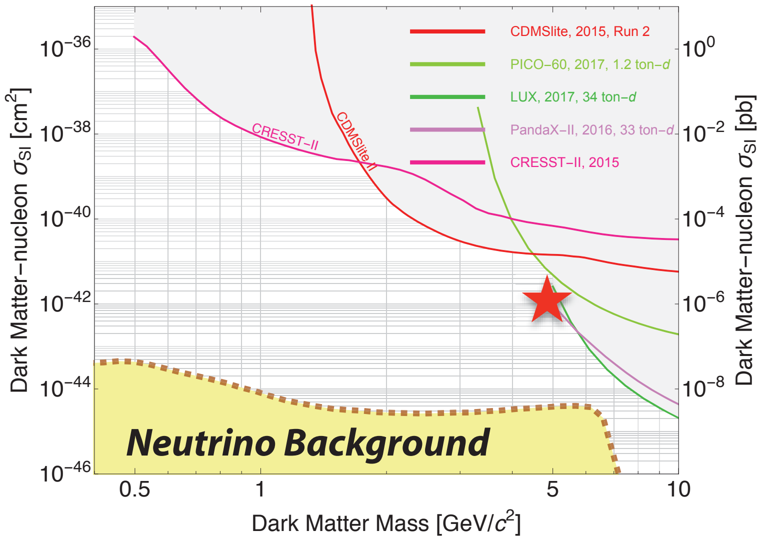
We cannot use our results based on the isolated atoms for the valence electrons.

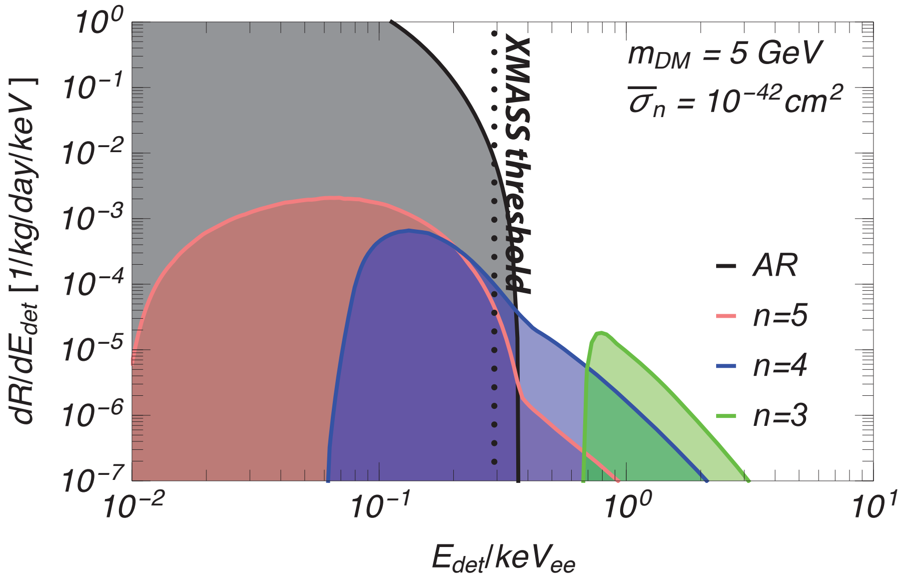
For the inner electrons, the effects from the environments are not significant.

✔ Implication on Dark Matter Direct Detection Experiments

✔ Migdal Effect single-phase Liquid Xe detectors

WIMP Limit Plotter





$$\frac{dR}{dE_{\rm det}} \simeq \left.\frac{dR}{dE_{\rm det}}\right|_{\rm w/o\ Migdal} + \left.\frac{dR}{dE_{\rm det}}\right|_{\rm w/\ Migdal}$$

$\boldsymbol{E_{det} = \underline{(0.1\text{-}0.2)}\, E_R + E_{EM}}$ $\boldsymbol{E_{EM} = E_e + E_{dex} \sim E_e - E_n}$

[Single phase Experiment = only scintillation energy :
Only **10-20 %** of $\boldsymbol{E_R}$ is measured]

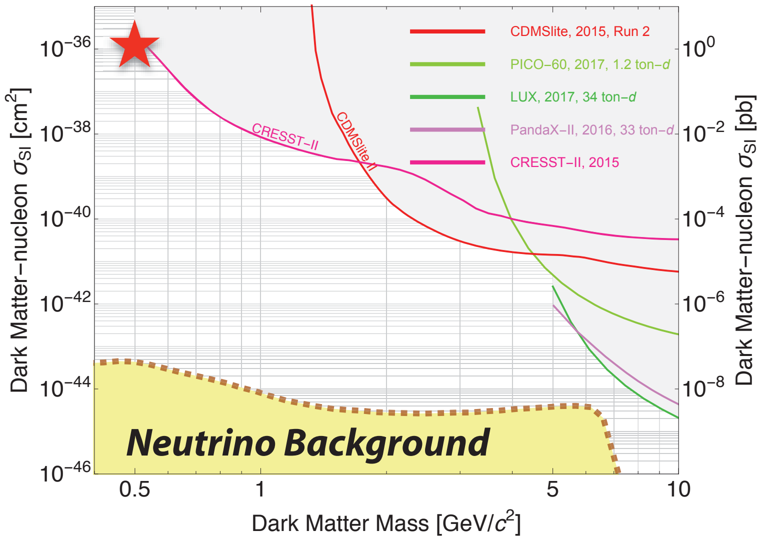
A few events with $E_{det} = O(1)$keV are expected for 10^5 kg days !

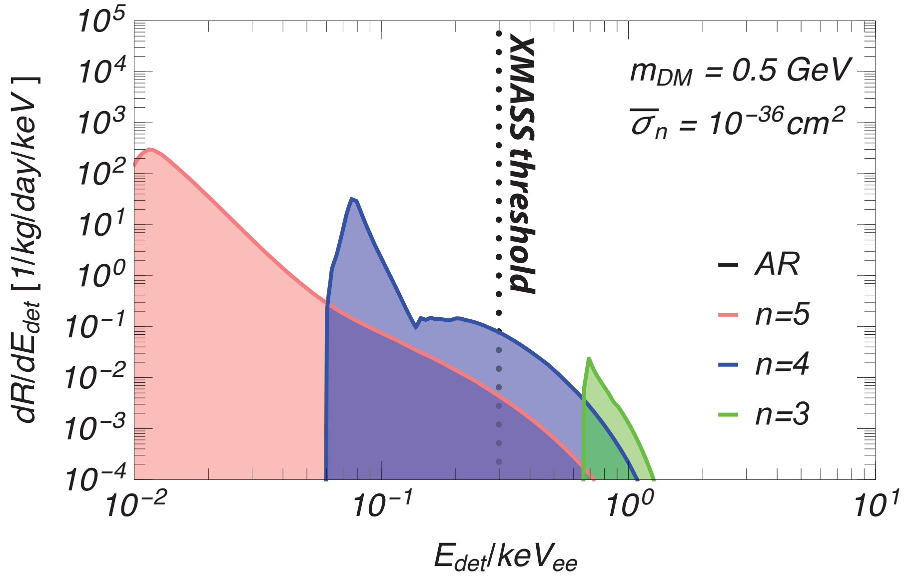
The atom recoil energy is lower than threshold $E_R < M_{DM}^2 / M_A \times v_{DM}^2 < O(1)$keV

Implication on Dark Matter Direct Detection Experiments

Migdal Effect single-phase Liquid Xe detectors

WIMP Limit Plotter





$$\frac{dR}{dE_{\rm det}} \simeq \left.\frac{dR}{dE_{\rm det}}\right|_{\rm w/o\ Migdal} + \left.\frac{dR}{dE_{\rm det}}\right|_{\rm w/\ Migdal}$$

$$E_{det} = (0.1\text{-}0.2)\, E_R + E_{EM} \qquad E_{EM} = E_e + E_{dex} \sim E_e - E_n$$

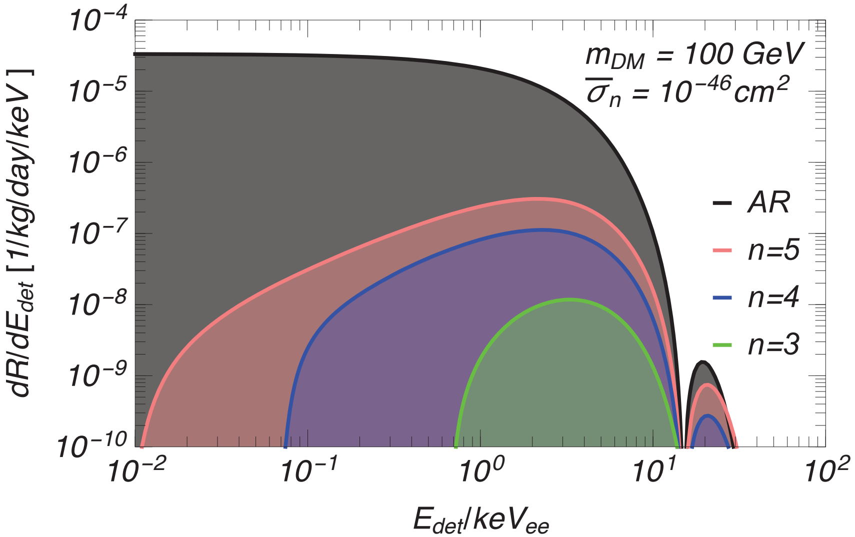
[Single phase Experiment = only scintillation energy :
Only **10-20 %** of E_R is measured]

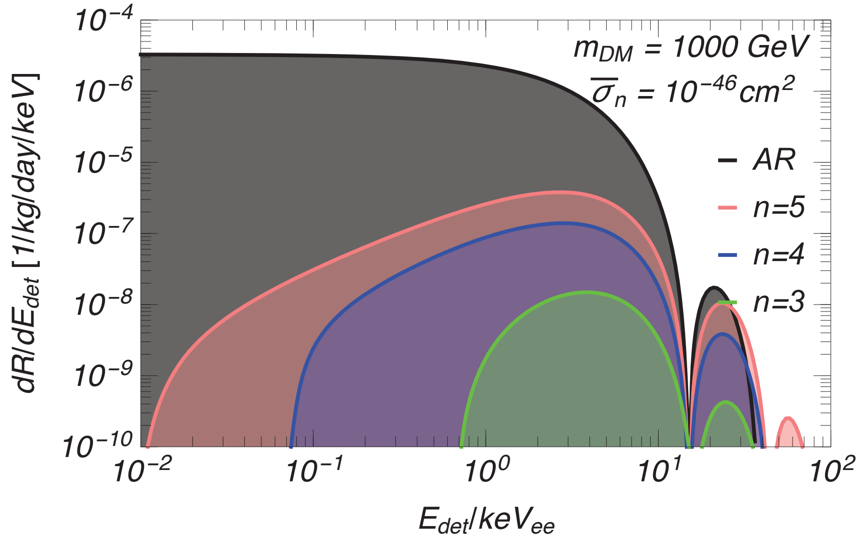
A few hundred events with E_{det} = O(1)keV are expected for 10^5 kg days !

The atom recoil energy is much lower than threshold $E_R < M_{DM}^2 / M_A \times v_{DM}^2$ = O(1)eV

Implication on Dark Matter Direct Detection Experiments

Migdal Effect single-phase Liquid Xe detectors





$$\frac{dR}{dE_{\rm det}} \simeq \left.\frac{dR}{dE_{\rm det}}\right|_{\rm w/o\,Migdal} + \left.\frac{dR}{dE_{\rm det}}\right|_{\rm w/\,Migdal}$$

$E_{det} = (0.1\text{-}0.2)\, E_R + E_{EM}$ $\quad E_{EM} = E_e + E_{dex} \sim E_e - E_n$

For heavier dark matter, the atom recoil energy is much lower than threshold

$E_R < M_A^2 \times v_{DM}^2 = O(10\text{-}100)\mathrm{keV}$

The Migdal effect is submerged below the conventional nuclear recoil spectrum.

✓ *SUMMARY*

- In the conventional analysis of dark matter direct detection experiments through the nuclear scattering, the whole atom is assumed to be recoiled.

- In reality, the electrons take some time to catch up with the recoiled nucleus leading to electronic energy injection in addition to the atomic recoil → Migdal Effect

- We reformulated the Migdal effect, where we can manifestly see the energy-momentum conservation and the probability conservation.

- The emitted electronic energy can be in the ***keV*** range even for a rather light dark matter ($\boldsymbol{M_{DM} < 10GeV}$) where the atomic recoil energy is lower than energy threshold, i.e. ***O(1)keV***.

- ***Migdal Effects has advantages to look for small "q" with a large cross section dark matter → Lower Mass dark matter such as SIDM/Asymmeteric Dark matter***

- ***Experimental confirmation is important !!!***