Energy partition and distribution of excited species in direction-sensitive detectors for WIMP searches

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Key words:
$S_T = S_e + S_n$
quenching factor $q_{nc} = \eta/E$
LET$_{el} = d\eta/dx$
Bragg-like curve: $d\eta/dR_{PRJ}$

Don’t miss ISON in late this year
Head–tail discrimination

The Bragg-like curve for head-tail detection

The distribution of the electronic energy $\eta$ deposited in the detector gas as a function of the ion depth, i.e. projected range $R_{PRJ}$.

$\Delta \eta / \Delta R_{PRJ}$

It is an averaged one dimensional presentation. For slow ions, it is not given by the electronic stopping power, $S_e = (dE/dx)_e$. One needs the Lindhard factor $q_{nc} = \eta/E$. 

Fig. 10. Sketch illustrating definition of range concepts $R$, $R_p$, $R_c$ and $R_L$. 
Nuclear Stopping Power

Interaction potential

\[ U(r) = (Z_1 Z_2 e^2 / r) \cdot \phi(r / a) \]

\[ \phi(r / a) \text{: Fermi function} \]

\[ a = 0.8853 \cdot a_0 \left( Z_1^{2/3} + Z_2^{2/3} \right)^{-1/2} \]

A screened Rutherford scattering

\[ \Rightarrow \exp(-r/a_B): \text{Bohr} \]

\[ a: \text{Thomas-Fermi type screening radius} \]

A universal differential cross section

\[ d\sigma = \frac{\pi a^2}{2t^{3/2}} f(t^{1/2}) \]

\[ t = \varepsilon^2 \cdot \left( \frac{T}{T_m} \right) = \varepsilon^2 \sin^2 \frac{\theta}{2} \]

The nuclear stopping power

\[ \left( \frac{d\varepsilon}{d\rho} \right)_n = \int_0^\varepsilon dx \frac{f(x)}{\varepsilon} \]

\[ f(t^{1/2}) \quad \text{Lindhard} \]

\[ t^{1/2} = \varepsilon \sin(\theta/2) \]
Stopping Powers

The nuclear stopping power $S_n$

$$S_n(E) = \frac{\langle T(E) \rangle}{\lambda(E)} = N\sigma < T > = N\sigma \int T(E, \theta(p)) \frac{2\pi p dp}{\sigma} = \pi N \int_0^\infty T d(p^2)$$

$\langle T(E) \rangle$ is the mean energy transferred in an elastic collision, and $\lambda(E) = 1/N\sigma$

$$T(E, \theta) = \frac{4A_1 A_2}{(A_1 + A_2)^2} E \sin^2 \theta \frac{\theta}{2}$$

$S_n$ can be expressed by the analytical expression [Birsack 1968]

$$S_n = -\frac{dE}{ds} = \frac{4\pi a N A_1 Z_1 Z_2 e^2}{A_1 + A_2} \cdot \ln \epsilon \cdot \frac{\ln \epsilon}{2\epsilon (1 - \epsilon^{-3/2})}$$

for all $Z_1, Z_2$

The electronic stopping power $S_e$

An atom moving though an electron gas of constant density. Using a Thomas–Fermi treatment, [Lindhard & Scharff]

$$S_e = \xi_e \times 8\pi e^2 a_0 \cdot \frac{Z_1 Z_2}{(Z_1^{2/3} + Z_2^{2/3})^{3/2}} \cdot \frac{v}{v_0}$$

$$\xi_e \approx Z_1^{1/6}$$

for all $Z_1, Z_2$

$$S_e = k\epsilon^{1/2}: 0.1 \sim 0.2$$
**Stopping Power**

Lindhard

Low energy

\[ v < v_0 = \frac{e^2}{\hbar} \]

The generalized range and energy

\[ \rho = RNA_2 \cdot 4\pi a^2 \frac{A_1}{(A_1 + A_2)^2} \]

\[ \varepsilon = E \frac{aA_2}{Z_1Z_2e^2(A_1 + A_2)} \]

Nuclear \( S_n \) and electronic \( S_e \) stopping powers as a function of energy \( \varepsilon \) for \( k=0.15 \).

The Thomas-Fermi treatment becomes a crude approximation at the extreme low energy and separation of the nuclear scattering and electronic stopping becomes uncertain. LSS theory is not very reliable at \( \varepsilon < 0.01 \), i.e., below 10 keV for Xe ions in Xe.

In this sense, almost all the stopping theory available is not reliable below \( \varepsilon < 0.01 \).
Lindhard factor \( q_{nc} = \frac{\eta}{\varepsilon} \)

The stopping powers contain only a part of the necessary information to obtain the quenching factor, \( q_{nc} = \frac{\eta}{\varepsilon} \) ratio. The differential cross section in nuclear collisions is needed for the integral equations. For \( Z_1 = Z_2 \),

\[
\left( \frac{d\varepsilon}{d\rho} \right)_{\varepsilon} \cdot \nu'(\varepsilon) = \int_{\varepsilon^2}^{\varepsilon} \frac{dt}{2t^{3/2}} \cdot f(t^{1/2}) \left\{ \nu \left( \frac{\varepsilon - t}{\varepsilon} \right) - \nu(\varepsilon) + \nu \left( \frac{t}{\varepsilon} \right) \right\}
\]

\[
\left( \frac{d\varepsilon}{d\rho} \right)_{\varepsilon} = k \varepsilon^{1/2}
\]

\( \eta \) as a fn of \( \varepsilon \) for \( k = 0.2, 0.15, 0.1 \)

\( \varepsilon = \eta + \nu \)
Electronic Linear Energy Transfer ($\text{LET}_{el}$)

$\text{LET}_{el} \equiv -d\eta/dR = -\Delta \eta/\Delta R$  

$R$: the range  

$= -(\eta_1 - \eta_0)/(R_1 - R_0)$  

for quenching calc. etc.

The true range $R$ is given by the total stopping power  

$R_T = \int (dE/dx)_{\text{total}}^{-1} \, dE$

The Bragg-like curve for TPC  

The projected range, $R_{PRJ}$, may be used

Birks’ eq.  

$$\frac{dL}{dx} = \frac{C_1(-dE/dx)}{1 + C_2(-dE/dx)}$$

LET$_{el}$ should be used, not $S_{el}$. 

- $R_{PRJ}$ (depth)
Fig. 1  The stopping power and the electronic LET as a function of the recoil energy for Xe in Xe.

HMI & Lindhard
Bragg-like curves, $d\eta/dR_{PRJ}$ for recoil ions in rare gases.

The ions enter from the right hand side. Points are plotted at every 5, 10 or 20 keV. The projected ranges are taken from SRIM.

Head and tail detection of WIMPs
He in He (+5%$C_4H_{10}$)

Ionizing particle produces excited atoms $He^*$ and ions $He^+$.
Then formation of excited molecules,
$$He^* + He + He \rightarrow He_2^* + He$$

molecular ions,
$$He^+ + He + He \rightarrow He_2^+ + He$$

and higher excited states
$$He^+ + e^- + He \rightarrow He_2^* + He$$

Hornbeck-Molnar Process
$$He^*(n \geq 2) + He \rightarrow He_2^- + e^-$$

Penning ionization
$$He^* + M \rightarrow He + M^+ + e^-$$
$$He_2^* + M \rightarrow He + He + M^+ + e^-$$

There are passes to no ionization
$$He^* + M \rightarrow He + M$$
(excitation, dissociation, non radiative decay)

The measured points are considerably smaller than Lindhard and SRIM.

The W-value can depend on He pressure and dopant pressure.
W-value

The energy balance of absorbed energy $T$ and $W$ value:

$$T_0 = \frac{N_i}{E_i} + \frac{N_{ex}}{E_{ex}} + N_i \bar{\epsilon}$$

$$W = \frac{T_0}{N_i} = \frac{E_i}{E_{ex}(N_{ex}/N_i) + \bar{\epsilon}}$$

$W_\alpha/W_\beta \approx 1$ for rare gases fast ions

Contributions of $2^1S$ and $2^3S$ may be added for slow ions in He.

He($2^1S$), He($2^3S$) and He$_2$(2$^1S$) and He$_2$(2$^3S$) are ‘true’ metastables.

He($2^1S$) and He($2^3S$) were not produced directly by fast particles; however can be produced directly by slow ions.

Resonance trapping for He($2^1P$).

(low pressure)

A part of energy will diffused out of the chamber.

The $W$-value can depend on energy and type of the incident particle at low energy.
Lindhard factor for $Z_1 \neq Z_2$

The formation of accurate general solutions become quite complicated. The integral equation for the ion $Z_1$ in the matter $Z_2$ is,

$$\nu_1'(E) \cdot S_{1e} = \int d\sigma_1 \left\{ \nu_1(E - T) - \nu_1(E) + \nu(T) \right\}$$

$d\sigma_1$: the diff. cross section for an elastic colli. between $Z_1$ and $Z_2$.

$$T < T_m = \gamma E ; \quad \gamma = 4A_1A_2/(A_1 + A_2)^2$$

Characteristic energies:

$$E_{1c} \equiv A_1^3(A_1 + A_2)^{-2} Z_1^{4/3} Z_1^{-1/3} \cdot 500 \text{ eV}, \quad E_{2c} \equiv (A_1 + A_2)^2 A_1^{-1} Z_2 \cdot 125 \text{ eV}, \quad \text{where } Z^{2/3} = Z_1^{2/3} + Z_2^{2/3}$$

The power law approximation for very low energy,

$$\eta = CE^{3/2}$$

$$C = \frac{2}{3} \left\{ E_{1c}^{-1/2} + \frac{1}{2} \gamma^{1/2} E_{1c}^{-1/2} \right\}$$

for $E < E_{1c}, E_{2c}$.

The straggling:

$$\frac{\Omega^2}{\eta^2} = \frac{1}{14}\gamma \left\{ \left( \frac{\gamma^{1/2}}{CE_{1c}^{1/2}} - \frac{7}{4} \right) + \frac{7}{16} \right\}$$

Good for heavy ions such as Pb ions in $\alpha$-decay
Lindhard factor for recoil ions in $\alpha$-decay

<table>
<thead>
<tr>
<th>Recoil ion</th>
<th>206Pb</th>
<th>208Tl</th>
<th>208Pb</th>
<th>$E_2c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy keV</td>
<td>103</td>
<td>117</td>
<td>168</td>
<td></td>
</tr>
<tr>
<td>gas</td>
<td>expt</td>
<td>calc</td>
<td>expt</td>
<td>calc</td>
</tr>
<tr>
<td>Ar</td>
<td>0.221a</td>
<td>0.196</td>
<td>0.263</td>
<td>0.205</td>
</tr>
<tr>
<td>Xe</td>
<td>0.139a</td>
<td>0.124</td>
<td>0.132</td>
<td>0.158</td>
</tr>
<tr>
<td>H2</td>
<td>0.73</td>
<td>0.457</td>
<td>0.78</td>
<td>0.544</td>
</tr>
<tr>
<td>He</td>
<td>0.53</td>
<td>0.500b</td>
<td>0.56</td>
<td>0.546b</td>
</tr>
<tr>
<td>CH4</td>
<td>0.250</td>
<td>0.265</td>
<td>0.294</td>
<td></td>
</tr>
<tr>
<td>C2H4</td>
<td>0.236</td>
<td>0.269</td>
<td>0.321</td>
<td></td>
</tr>
<tr>
<td>C3H6</td>
<td>0.272</td>
<td>0.281</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>0.336</td>
<td></td>
<td>0.347</td>
<td></td>
</tr>
<tr>
<td>C + 2O</td>
<td>0.297</td>
<td>0.316</td>
<td>0.378</td>
<td>174</td>
</tr>
<tr>
<td>C</td>
<td>0.323</td>
<td>0.344</td>
<td>0.411</td>
<td>174</td>
</tr>
<tr>
<td>O</td>
<td>0.284</td>
<td>0.302</td>
<td>0.361</td>
<td>241</td>
</tr>
<tr>
<td>N2</td>
<td>0.319</td>
<td>0.320</td>
<td>0.384</td>
<td>207</td>
</tr>
<tr>
<td>Dry air</td>
<td>0.296</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4N + O</td>
<td>0.298</td>
<td>0.317</td>
<td>0.379</td>
<td>207</td>
</tr>
<tr>
<td>CS2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C + 2S</td>
<td>0.246</td>
<td>0.262</td>
<td>0.314</td>
<td>174</td>
</tr>
<tr>
<td>S</td>
<td>0.208</td>
<td></td>
<td></td>
<td>550</td>
</tr>
<tr>
<td>Al (for CS2)</td>
<td>0.228</td>
<td></td>
<td></td>
<td>428</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Recoil ion</th>
<th>214Pb</th>
<th>210Pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy keV</td>
<td>112</td>
<td></td>
</tr>
<tr>
<td>CS2</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>C + 2S</td>
<td>0.346</td>
<td>0.292</td>
</tr>
<tr>
<td>Al (for CS2)</td>
<td>0.237</td>
<td>0.272</td>
</tr>
</tbody>
</table>

**calc**: the power law approximation by Lindhard which is only good at $E<E_2c$.

**expt**: $W(\alpha)/W(recoil)$ from:

Binary gases

Binary gases such as $\text{CS}_2$, $\text{CF}_4$

$S_T$ the Bragg rule

compound correction

Simple approximation for $q_{nc}$ to consider only one element at a time.

$C$ in $\text{CS}_2 \Rightarrow C$ in $C$,

$S$ in $\text{CS}_2 \Rightarrow S$ in $S$

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solid: $C$ ions in $C$, $S$ ions in $S$

dashed: Asymptotic eq. $C$ and $S$ in Al

●, ●: Snowdon-Ifft expt.

Power law approximation

$1/3(\text{Pb in C})+2/3(\text{Pb in S})$
$[C] : [F] = 1:4$
The maximum impact parameter $b_{\text{max}}$ is given by Bohr’s impulse principle

$$b_{\text{max}} = \frac{\hbar v}{2E_1}$$

$v$: the incident ion velocity

$\hbar$: Planck’s constant divided by $2\pi$

$E_1$: the lowest excitation energy.

Excitation by the light ions ($p, \alpha$) described theoretically in terms of perturbation by an incident point charge.
The Coulomb effect
the deflection and deceleration of the projectile
in the field of the nucleus.

The Threshold effect
the energy delivered to $e^-$ must be as large as $I$.

Stopping power for slow light particles

I.S. Tilinin, PRA (1995)
H-C (1)
He-N (2)
The measurement of stopping power for heavy ions of low velocity is quite difficult. Usually, \( S_T = S_e + S_n \) is measured and often Lindhard is assumed for \( S_e \).

A. Fukuda,


The spectrometer determines the energy quite accurately; however, the measurement does not take contribution from \( \theta > 0 \).
When atomic projectile goes hard (wide deflection angle small impact parameter) collision with atom, the large inelastic energy losses occur at characteristic internuclear distances. Showers of fast-electrons are thrown out.

<table>
<thead>
<tr>
<th>$T_0,\theta$</th>
<th>$m,n$</th>
<th>$\bar{Q}_{mn}$ (eV)</th>
<th>$m,n$</th>
<th>$\bar{Q}_{mn}$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 keV, 8°</td>
<td>$T,T$</td>
<td>57±3</td>
<td>1,1</td>
<td>55±3</td>
</tr>
<tr>
<td></td>
<td>0,1</td>
<td>36±3</td>
<td>2,1</td>
<td>79±4</td>
</tr>
<tr>
<td></td>
<td>1,0</td>
<td>30±4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 keV, 16°</td>
<td>$T,T^a$</td>
<td>90±17</td>
<td>2,2$^b$</td>
<td>353±7</td>
</tr>
<tr>
<td></td>
<td>$T,T^b$</td>
<td>379±10</td>
<td>2,3$^b$</td>
<td>362±9</td>
</tr>
<tr>
<td></td>
<td>$T,T^c$</td>
<td>613±14</td>
<td>2,3$^c$</td>
<td>636±14</td>
</tr>
<tr>
<td></td>
<td>1,1$^a$</td>
<td>62±6</td>
<td>3,3$^b$</td>
<td>468±6</td>
</tr>
<tr>
<td></td>
<td>2,2$^a$</td>
<td>160±7</td>
<td>3,3$^c$</td>
<td>647±10</td>
</tr>
</tbody>
</table>

Kessel & Everhart, PR146 (1965)
Molecular orbital (MO) collision theory

$v_c << v_e$, the electronic motion of the system is expected to adjust adiabatically to the changing position of colliding nuclei.

→ The transient formation of molecular orbitals (MO’s)

MO’s can connect different shells. An electron occupies a MO and become excited during the collision to a higher energy at smaller separation. It contains vacancies after the collision.

Coordinate system for a quasimolecule composed of an electron $e^-$ and two nuclei $A$ and $B$.

J. Eichler, Lectures on ion-atom collisions
Fig. 2. Crossing of potential curves. Two potential curves for the states $s'$ and $s$ may cross in a certain approximation (such as in a single configuration molecular orbital theory). In a higher approximation, the curves repel each other. If the atoms approach each other slowly in state $s$, an adiabatic transition from $s$ to $s'$ will occur. If they approach each other rapidly, a diabatic transition from $s$ to $s$ will occur.

W. Lichten, PR 131 (1963)
Nuclear quenching factor

\[ q_{nc} = \eta/\varepsilon \text{ ratio} \]

Some puts:

\[ q_{nc}(\varepsilon) = \frac{\varepsilon \cdot S_{el}(\varepsilon)}{S_{nc}(\varepsilon) + S_{el}(\varepsilon)} \]  

RHS: integrated value  
LHS: differential value
and insight. The voluminous literature that has accumulated since then attests to the unfortunate fact that experimental determination of $W$ is often very easy, but accurate determination is not. This literature abounds in disparities and contradictions, and even the greatly augmented activity of the past decade has failed to resolve many of them. It is a pity that published tables of experimental data do not bear a symbol indicating whether or not the data are correct.


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Wave optics using ultrasound

The wave $U_0$ observed without deflecting object delays $\pi/2$ in phase compared to the wave $U_{sa}$ passing on the central axis.

Books by
Born&wolf, Jenkins&White, etc.